HEAT AND MASS TRANSFER TO PARTICLES DURING SEPARATION-FREE JET FLOW AND INTERNAL TRANSFER IN A GRANULAR BED

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Convective heat or mass transfer under jet flow conditions is of interest in connection with problems of cooling solid surfaces, the organization of a number of mass transfer processes in chemical technology apparatus, etc. Moreover, the model of separation-free flow around particles by axisymmetric jets is successfully relied upon at this time for explanation of regularities of heat and mass transfer to particles of fixed granular beds that is important to a broad circle of applications [1, 2].

The transfer to particles around which jets flow has been investigated repeatedly both experimentally and theoretically [3-8]. The quantity of investigations of interphasal transfer in granular beds is exceptionally high (see the bibliography in [2], for example). Nevertheless, up to now there has been no total clarity about the nature of the dependence of the appropriate Sherwood Sh (or Nusselt) parameter on the Reynolds number Re. The correlation of experimental data usually results in power-law criterial dependences with substantially different exponents, where the discrepancy between the appropriate empirical formulas reaches an order and higher.

The problem is examined in this paper for large Schmidt Sc numbers (or Prandtl) for spherical particles around which a laminar axisymmetric jet flows separation-free. It is shown that the dependence Sh(Re) cannot be approximated by a simple power-law formula in a broad range of Re variation.

1. Let us examine the flow around a sphere of radius a by a cylindrical jet of incompressible fluid of radius r_0 with a velocity u_0; the flow pattern and the coordinates introduced are represented in Fig. 1. The flow at the sphere surface consists of three zones: the film flow domain for $\theta_0 < \theta < \theta_*$ and the domains near the stagnation points $\theta = 0$ and $\theta = \pi$.

The characteristics of the laminar boundary layer being formed in the frontal domain can be found, in principle, by using the Froessling method [9] if the solution of the problem about ideal fluid jet flow around a sphere is used as the appropriate external asymptotic expansion. In any case, in direct proximity to the forward stagnation point it is allowable to use the formula

$$u_s = \frac{3}{2}\alpha\left(\frac{3\,\mathrm{Re}}{2}\right)^{1/2} u_0\theta\,\frac{y}{a} \approx \frac{3}{2}\alpha\left(\frac{3\,\mathrm{Re}}{2}\right)^{1/2} u_0 \sin\theta\,\frac{y}{a},\quad \mathrm{Re} = \frac{2au_0}{\nu}, \tag{1.1}$$

for the velocity near a solid surface, where ν is the kinematic viscosity; α is a numerical coefficient equal to 0.9277 for the unlimited homogeneous flow around a sphere. In a first approximaton this same value of α can also be used for jet flow, as is confirmed by the independence of the heat or mass flux at a frontal point from the incident jet diameter established in test [8].

Film flow on a spherical surface is analyzed in [2]. Since it contains serious inaccuracies, we examine this flow in detail. The Bol'ttse equations for the flow in a thin spherical layer have the form [9]

$$\partial(\rho u)/\partial x + \partial(\rho v)/\partial y = 0,\ u\partial u/\partial x + v\partial u/\partial y = \nu\partial^2 u/\partial y^2,$$

$$\rho \approx a\sin\theta,\ x = a\theta,\ y = r - a.$$

Integrating them with respect to dy between 0 and δ [$\delta(x)$ is the liquid film thickness] and taking account of the boundary conditions $u = v\ 0$, $y = 0$; $\partial u/\partial y = 0$, $y = \delta$, we obtain the relationships

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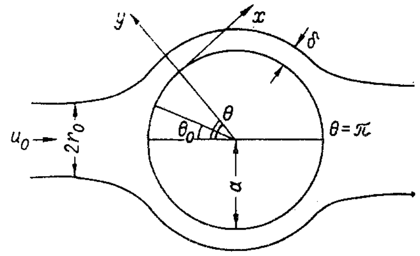

Fig. 1

$$\int_0^\delta \frac{\partial}{\partial x}(\rho u)\,dy = -\rho U \frac{d\delta(x)}{dx}, \tag{1.2}$$

$$\int_0^\delta \frac{\partial}{\partial x}(\rho u^2)\,dy + \rho U^2 \frac{d\delta(x)}{dx} = -\nu\rho \left.\frac{\partial u}{\partial y}\right|_{y=0},$$

where U is the tangential velocity component on the free film surface [it is taken into account that $v = U d\delta(x)/dx$ on this surface].

Using the Kármán–Pohlhausen method as in [2], we take

$$u = U((3/2)\eta - (1/2)\eta^3),\ \eta = y/\delta. \tag{1.3}$$

The equations

$$\frac{d}{d\theta}(U\delta\sin\theta) = 0,\ \ \frac{17}{35}\frac{d}{d\theta}(U^2\delta\sin\theta) = -\nu a \sin\theta\,\frac{3}{2}\frac{U}{\delta}. \tag{1.4}$$

follow from (1.2) and (1.3). Introducing the bulk fluid flow rate in the medium $G = \pi r_0{}^2 \times u_0 = 2\pi\, a\sin\theta \int_0^\delta u\,dy$, we find from (1.4)

$$\delta = \frac{4}{5\pi}\frac{G}{a}\frac{1}{U\sin\theta},\quad \frac{1}{U^2}\frac{dU}{d\theta} = -\frac{2625}{544}\frac{\pi^2 a^3 \nu}{G^2}\sin^2\theta. \tag{1.5}$$

The solution of (1.5) can be represented in the form

$$U = U_0\left[1 + \frac{2625}{544}\frac{\pi^2 a^3 \nu}{2G^2}\left(\theta - \theta_0 + \frac{1}{2}\sin 2\theta_0 - \frac{1}{2}\sin 2\theta\right)\right]^{-1}, \tag{1.6}$$

where U_0 is the value of U for $\theta = \theta_0 = \arcsin(r_0/a)$. Introducing Re in conformity with (1.1) and utilizing (1.3), (1.5), and (1.6), we obtain for the velocity u_s at the surface

$$u_s = \frac{3}{2}U\frac{y}{\delta} = \frac{15\pi}{8}\cdot\frac{a^2U^2}{G}\sin\theta\,\frac{y}{a} = \frac{15}{8}\left(\frac{a}{r_0}\right)^2\left[K + \frac{2625}{544\,\mathrm{Re}}\left(\frac{a}{r_0}\right)^4\left(\theta - \theta_0 + \frac{1}{2}(\sin 2\theta_0 - \sin 2\theta)\right)\right]^{-2} u_0 \sin\theta\,\frac{y}{a} \tag{1.7}$$

[$K = u_0/U_0$ can be found from the condition of consistency of the expressions (1.1) and (1.7) at $\theta = \theta_0$]. Hence

$$K = \left(\frac{5}{4\alpha}\sqrt{\frac{2}{3}}\right)^{1/2}\frac{a}{r_0}\,\mathrm{Re}^{-1/4}. \tag{1.8}$$

Therefore, it is impossible to consider the assumption $U_0 = u_0$ taken in [2] as well as in a number of other papers justified. The velocity on the film-free surface increases in proportion to $u_0{}^{5/4}$ as the jet velocity grows and depends strongly on the jet radius (the latter is easily understood by taking account of the proportionality of the flow rate in the jet to the area of its transverse section).

Let us note that the most simple procedure used above to connect the flows in the frontal domain and the film flow domain results in the appearance of a break in the dependence of u_s on θ. In principle it is easy to improve this procedure (and, in particular, to eliminate the mentioned break) if the next terms of the Froessling series that govern their coefficients from the conditions of continuity of the derivatives of u_s with respect to θ, are introduced into (1.1).

Introducing the stream function ψ for flows near the solid surface, we obtain from (1.1) and (1.7)

$$\psi = u_0 y^2 g(\theta),\ \ u_s = \frac{1}{r\sin\theta}\frac{\partial\psi}{\partial y} \approx \frac{1}{a\sin\theta}\frac{\partial\psi}{\partial y}, \tag{1.9}$$

where the function $g(\theta)$ and the characteristic Reynolds number have been introduced

$$g(\theta)=\begin{cases}0.852\sqrt{\mathrm{Re}}\sin^2\theta, & 0<\theta<\theta_0,\\ 0.852\sqrt{\mathrm{Re}}\left[1+\left(\frac{\mathrm{Re}_*}{\mathrm{Re}}\right)^{3/4}\left(\theta-\theta_0+\frac{1}{2}\sin 2\theta_0-\frac{1}{2}\sin 2\theta\right)\right]^{-2}\sin^2\theta, & \theta_0<\theta<\pi,\end{cases} \tag{1.10}$$

$$\mathrm{Re}_*=\left[\frac{2625}{544}\left(\frac{4\alpha}{5}\sqrt{\frac{3}{2}}\right)^{1/2}\right]^{4/3}\left(\frac{a}{r_0}\right)^4\approx 9.64\left(\frac{a}{r_0}\right)^4$$

(the Reynolds number value corresponds to $\alpha = 0.9277$).

The flow in the root domain can also be described by considering a small neighborhood of the appropriate stagnation point. However, there is practically no necessity in the investigation of converse transfer since the heat and mass transfer in the root part introduce quite an insignificant contribution to the total heat or mass fluxes to the sphere. Moreover, there is no assurance that at least a small vortex zone is not formed near the root stagnation point in reality since the assumption about the flow being separation-free should be considered approximate. Consequently, it is allowable to extend the film flow domain down to $\theta = \pi$, as has been done above.

2. For definiteness, we consider convective diffusion to an ideally absorptive particle. For $\mathrm{Sc} = \nu/D \gg 1$ (D is the diffusion coefficient of the impurity being absorbed), the approximation is valid of a thin diffusion layer in which the flow is described completely by the relationships (1.9) and (1.10). Inserting the Mises variables in the usual manner and solving the self-similar convective diffusion problem being obtained by using standard methods [10], we write

$$c=\frac{c_0}{1{,}17}\int_0^z\exp\left(-\frac{4}{9}t^3\right)dt,\quad z=\left(\frac{0{,}852}{4}\right)^{1/4}\mathrm{Sc}^{1/3}\,\mathrm{Re}^{1/2}f(\theta)\left(\int_0^\theta f(t)\sin t\,dt\right)^{-1/3}\frac{y}{a}, \tag{2.1}$$

where the function

$$f(\theta)=\begin{cases}\sin\theta, & 0<\theta\leqslant\theta_0=\arcsin(r_0/a),\\ \sin\theta\left[1+\left(\frac{\mathrm{Re}_*}{\mathrm{Re}}\right)^{3/4}\left(\theta-\theta_0+\frac{1}{2}\sin 2\theta_0-\frac{1}{2}\sin 2\theta\right)\right]^{-1}, & \theta_0<\theta\leqslant\pi.\end{cases} \tag{2.2}$$

has been introduced.

The mass flux density evaluated from (2.1) is

$$q=0.510\,\mathrm{Sc}^{1/3}\,\mathrm{Re}^{1/2}f(\theta)\left(\int_0^\theta f(t)\sin t\,dt\right)^{-1/3}\frac{Dc_0}{a}. \tag{2.3}$$

Its value at the forward stagnation point

$$q_0=q|_{\theta=0}\approx 0.736\mathrm{Sc}^{1/3}\mathrm{Re}^{1/2}Dc_0/a \tag{2.4}$$

agrees with that evaluated in [11] but is somewhat higher than that determined in [12]. Integrating (2.3) over the surface of the sphere we obtain the total mass flux Q to the particle that can be described by using the criterial dependence

$$\mathrm{Sh}=\frac{2a\langle q\rangle}{Dc_0}=\frac{2a}{Dc_0}\frac{Q}{4\pi a^2}=0.510\,\mathrm{Sc}^{1/3}\,\mathrm{Re}^{1/2}\int_0^\pi f(\theta)\left(\int_0^\theta f(t)\sin t\,dt\right)^{-1/3}\sin\theta\,d\theta. \tag{2.5}$$

The integrals in (2.3) and (2.5) can be expressed in terms of known functions. However, it is apparently simpler to use numerical methods to find them because of the extreme awkwardness of these expressions.

Dependences of the flux density $q(\theta)$ referred to its value q_0 at a frontal point are represented in Fig. 2 for different $\sigma = (r/a)^2$ (the numbers on the curves) for $\mathrm{Re} = 10^3$. An analogous dependence for the domain of separation-free flow is displayed by the dashed line for unlimited homogeneous flow around a sphere obtained in [11]. On the whole, the mass flux distribution over the sphere surface becomes more uniform upon going over to jet flow. For sufficiently thin jets a reduction in the relative flux density on the front part of the sphere surface occurs as compared with a sphere in an unlimited flow, which

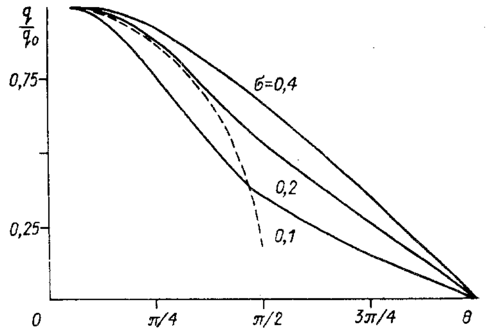

Fig. 2

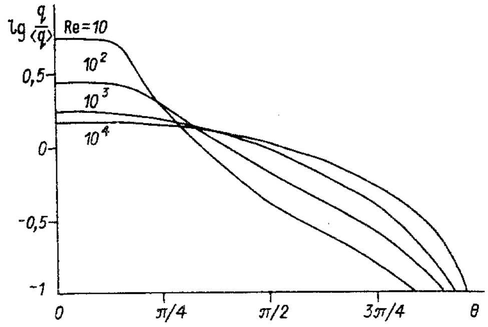

Fig. 3

is associated with the appropriate diminution in the velocity on the free liquid film surface. Refinement of the jet results in rapid growth of the critical number Re_* in (1.10) and (2.2).

Distribution of the relative mass flux density $q/\langle q\rangle$ for $\sigma = 0.2$ and different Re (number on the curves) is shown in Fig. 3. An increase in Re will result in essential equilibration of the flux density over the particle surface.

Dependences of the coefficient C(Re) on Re in the formula

$$\mathrm{Sh} = C(\mathrm{Re})\mathrm{Sc}^{1/3}\mathrm{Re}^{1/3} \tag{2.6}$$

following from (2.5) are given in Fig. 4 for different σ (numbers at the curves). The proportionality between Sh and $\mathrm{Re}^{1/2}$ is achieved only for large Re when $C(\mathrm{Re}) \simeq$ const. Such a dependence, found earlier in [6], follows asymptotically from (2.5) for $\mathrm{Re} \gg \mathrm{Re}_*$. Since $\mathrm{Re}_* \sim (a/r_0)^4$ [see (1.10)], then it is clear that the domain of Re in which the mentioned proportionality is disturbed should expand rapidly with refinement of the jet, as is seen in Fig. 4. It is possible to obtain Sh ~ Re from (2.5) for $\mathrm{Re} \ll \mathrm{Re}_*$ as is approximately valid in a range of Re variation all the broader, the thinner the jet. However, independently of the jet thickness the local Sherwood number determined for the forward stagnation point turns out to be proportional to $\mathrm{Re}^{1/2}$.

Each particle is flowed around by a jet in a flow in a granular bed, where the effective area of a section is approximately equal to the minimal through a section of a cell containing one particle [2]. If the porosity of the granular bed is $\varepsilon = 0.4$, then the cell section of $\sigma' = 0.172$, where $\sigma = \sigma'(1 - \sigma')^{-1} = 0.208$. The effective velocity of such a jet is $u_0 = u_f/\sigma' = u'(\varepsilon/\sigma')$ (u_f and u' are the filtration rate and the mean fluid velocity in the gaps between particles). If Re' is determined by means of these latter, then the Re introduced above is Re = 2.326 Re'. The dependence of Sh on Re' for $\sigma = 0.208$ that follows from (2.5) is displayed by the heavy lines in Fig. 5 where the empirical formulas (see the appropriate curves)

$$\mathrm{Sh}/\mathrm{Sc}^{1/3} = 0.529\mathrm{Re}'^{0,64}\ (1),\ 0.256\mathrm{Re}'^{0,7}\ (2),$$
$$0.0424\mathrm{Re}'\ (3),\ 0.21\mathrm{Re}'^{0,78}\ (4),\ 0.0575\mathrm{Re}'^{0,83}\ (5),$$

belonging to different authors and assembled in [2] are also presented. It is seen that the theoretical curve occupies an intermediate position among the empirical curves for all the Re' except the very largest. However, for high Re' turbulization of the laminar boundary layer and the flow in the liquid film can be expected and applicability of the developed theory becomes doubtful in the best case.

Keeping in mind applied purposes, it is reasonable to approximate the dependence (2.5) represented in Fig. 5 by a certain comparatively simple function. Using least squares, we can write

$$\lg(\mathrm{Sh}/\mathrm{Sc}^{1/3}) = -0.964 + 0.915\ \lg\mathrm{Re}' - 0.050(\lg\mathrm{Re}')^2. \tag{2.7}$$

in the interval $1 < \log \mathrm{Re}' < 5$ with a relative error not exceeding 1%.

It follows from an analysis of experimental data on internal transfer in granular beds that (2.7) is in good agreement with them. However, computations of the flux density distribution (2.3) over the particle surface for experiment conditions in [6, 8] resulted in

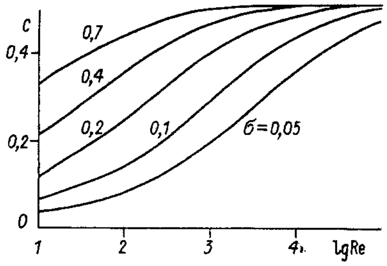

Fig. 4

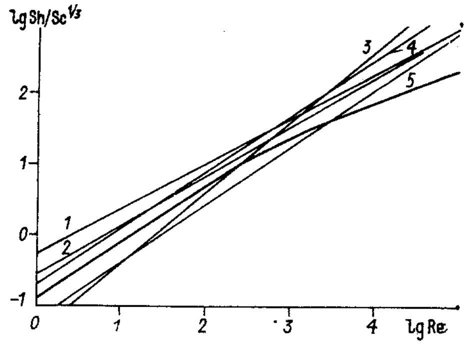

Fig. 5

values exceeding the experimental somewhat on the frontal hemisphere. The agreement between theory and experiment here is improved significantly when using values of σ in the computations that are smaller than those communicated in [8]. The discrepancy mentioned is apparently associated, first, with not taking account of natural jet contraction in experiments and, second, with the comparatively rough connection between the boundary-layer and film flows, as already noted above.

In conclusion, let us emphasize that the results obtained on the interphasal heat and mass transfer in a granular bed are referred to ordered beds in which porosity fluctuations can be neglected completely. The presence of inhomogeneities of a different linear scale is characteristic for real chaotically stacked beds. They all result in the appearance of fluid velocity fluctuations in the infiltrated beds, which are inevitably felt even in the interphasal transfer [2]. One of the problems of subsequent investigation can be discerned in an analysis of the influence of such fluctuations.

LITERATURE CITED

1. V. I. Volkov, N. S. Danilov, V. D. Zhak, et al., "Investigation of the hydrodynamics of a near-wall layer in a cubic stacking model," Zh. Prikl. Mekh. Tekh. Fiz., No. 6 (1980).
2. M. A. Gol'dshtik, "Transfer Processes in a Granular Bed [in Russian], Inst. Termofiz., Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1984).
3. H. Schuh and B. Persson, "Heat transfer on circular cylinders exposed to free-jet flow," Int. J. Heat Mass Transfer, 7, No. 11 (1964).
4. S. I. Isataev and Z. Zh. Zhanabaev, "Heat elimination of a sphere under jet flow," Inzh.-Fiz. Zh., 14, No. 4 (1968).
5. L. K. Vukovich, A. V. Nikolaev, and S. S. Titar', "Heat elimination of a sphere with internal heat liberation in a jet flow," Prom. Teplotekhnika, 4, No. 2 (1982).
6. V. V. Dektyareva, V. A. Mukhin, and V. E. Nakoryakov, "Mass transfer of a sphere from a heavy fluid jet," in: Jet Fluid and Gas Flows [in Russian], Abstracts of an All-Union Scientific Conference, Pt. 3, Novopolotsk (1982).
7. V. P. Kashcheev, A. V. Lebedev, and V. N. Sorokin, "Investigation of heat transfer between a single sphere and an air jet," Izv. Vyssh. Uchebn. Zaved. Énerg., No. 9 (1985).
8. V. A. Mukhin, Experimental Investigation of Transfer Processes in Granular Media [in Russian], Author's Abstract of Candidate's Dissertation, Inst. Teplofiz., Sib Otd. Akad. Nauk SSSR, Novosibirsk (1987).
9. H. Schlichting, Boundary-Layer Theory [translated from German], 6th ed., McGraw Hill, NY (1968).
10. V. G. Levich, Physicochemical Hydrodynamics [in Russian], Fizmatgiz, Moscow (1959).
11. Yu. A. Buevich and A. E. Shul'meister, "On heat or mass transfer to poorly-streamlined bodies," Inzh.-Fiz. Zh., 45, No. 2 (1983).
12. T. R. Galloway and B. H. Sage, "Thermal and material transfer from spheres. Prediction of local transport," Int. J. Heat Mass Transfer, 11, No. 3 (1968).